Test 3A - MTH 1410

Name: _____

PID Number: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

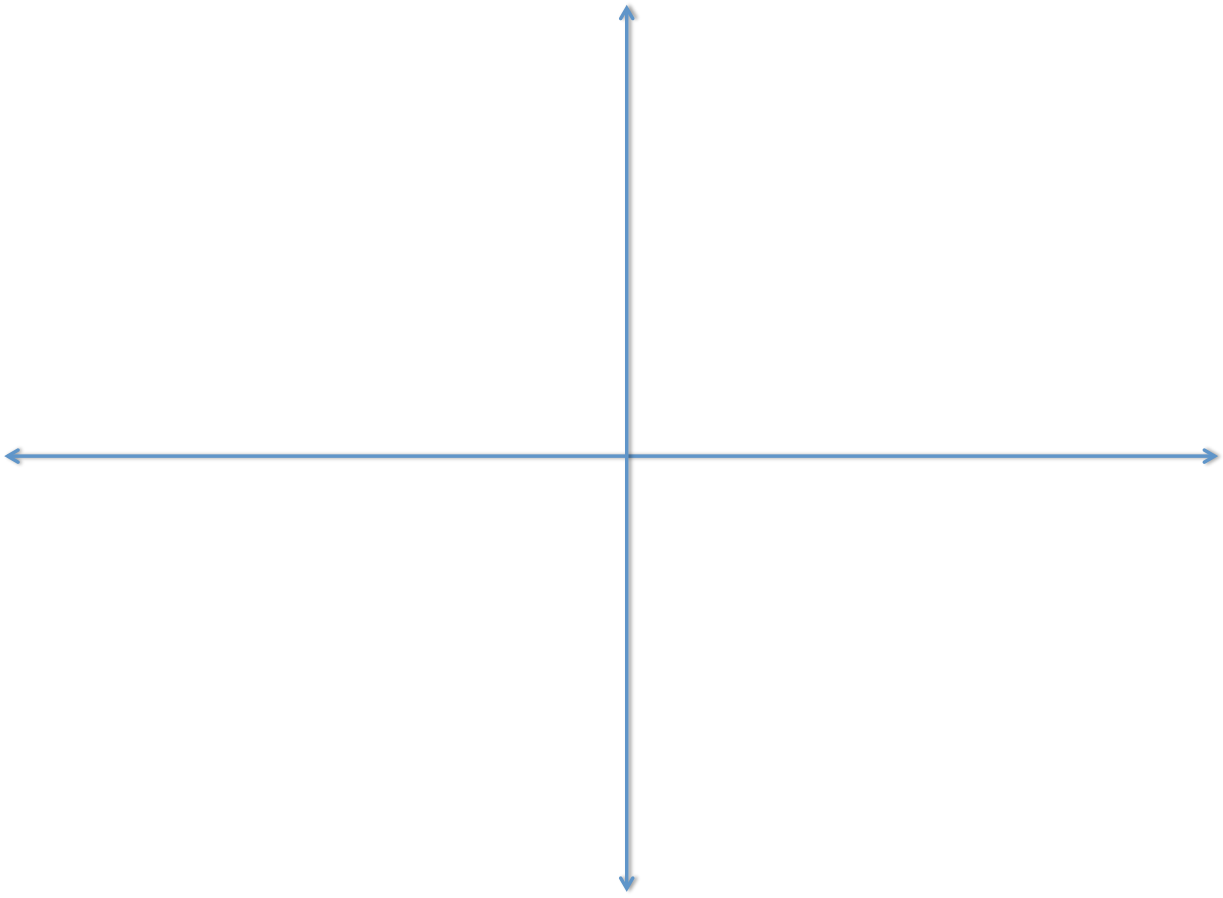
1. Show all of your work. A correct answer with insufficient work will be counted wrong.
2. Clearly indicate your answer by putting a box around it.
3. Calculators <u>are</u> allowed on this exam.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge and write your PID on both pages.
6. Number of questions = 10. Total Points = 100.

PID Number: _____

1. (12 points) Water is pouring into an inverted cone-shaped tank. The cone has a radius of 6 meters and a height of 4 meters. Note that the cone is inverted, so the base of the cone is actually the top of the tank. When the depth of the water is 2 meters, the depth is increasing at a rate of 0.2 meters/second. How fast is the volume of the water changing at that instant? (Note: The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, and you will need to use the fact that the radius equals 3/2 times the height).

2. (12 points) Sketch the graph of a <u>continuous</u> function $f(x)$ given the following information:

- $f(0) = 0$, the absolute minimum value of f is -4, and the absolute maximum value of f is 3.
- $f'(-3)$ and $f'(4)$ do not exist.
- $f'(x) > 0$ on the intervals $(-\infty, -3), (-3, -2)$ and $(2, 4)$.
- $f'(x) < 0$ on the intervals $(-2, 2)$ and $(4, \infty)$.
- $f''(x) > 0$ on the intervals $(-\infty, -3)$ and $(0, 4)$ and $(4, \infty)$.
- $f''(x) < 0$ on the interval $(-3, 0)$
- $\lim_{x \to (-\infty)} f(x) = -1$ and $\lim_{x \to \infty} f(x) = 0$



3. (6 points) Find the most general antiderivative of

$$f'(x) = \frac{x^6 - 2x^3 + 1}{x^4}$$

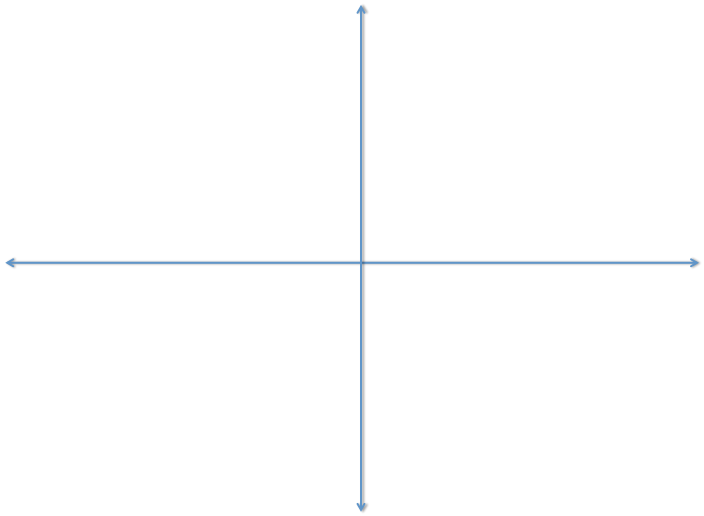
4. (12 points) Consider the function $f(x) = \dfrac{x}{3x^2 + 12}$.

(a) Find the x-coordinate(s) where f has a local maximum or a local minimum. Clearly identify if your answer is a local maximum or a local minimum, and show your work.

(b) Find the absolute maximum of f on the interval $[-4, 0]$.

5. (8 points) Two cars leave from the same point at the same time. One travels north at 21 miles/hour and the other travels east at 28 miles/hour. Use calculus to find the rate at which the two cars are moving away from each other after two hours.

6. (10 points) A rectangle with its bottom on the x-axis has upper corners on the parabola $y = 4.5 - \frac{1}{2}x^2$. Find the maximum <u>perimeter</u> possible for such a rectangle. Be sure to check that your answer is a maximum.



7. (8 points) For the following statements, answer True or False. If True, give a brief explanation of why. If False, either explain why or give a counterexample. A picture may help, but needs some words to explain it.

   (a) "Suppose I wanted to use Newton's Method on $f(x) = 2x^3 - 24x + 31$. Since $f(2) = -1$, $x_1 = 2$ would be a good first guess."

   (b) "For a continuous function defined on a closed interval, the absolute maximum and absolute minimum values must occur at either the endpoints of the interval or a critical number."

8. (10 points) On the surface of the moon, an observation tower is built with a catapult on top. The tower is 120 meters high, and the catapult shoots a ball off the side of the tower. Assuming acceleration due to gravity on the moon is a constant -4 m/sec^2, and the ball has initial upward velocity of 22 m/sec, how long will it take for the ball to hit the surface of the moon?

9. (12 points) Calculate the following limits. If the limit does not exist, explain why.

(a) $\lim_{x \to 0} \dfrac{\ln(1+x)}{\tan(2\pi x)}$

(b) $\lim_{x \to 0^+} \dfrac{e^x}{x^3 - 2x}$

10. (10 points) Given the equation $2x^2 - x = 3$, use Newton's Method to find a solution for x. Use a first guess of $x_1 = 1$ and find the solution accurate to the first two decimal places.

Extra Credit(2 points) Find an antiderivative of $f'(x) = e^e$.